Modeling and Characterization of Radial-mode Disk-type Piezoelectric Transformer for AC/DC Adapter

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Abstract— For characterization of the newly developed radial-mode disk-type piezoelectric transformer(PT) for ac/dc adapter applications, the closed-form solution of the wave equation, including a loss model for the PT design, was obtained without FEM tools and used to build the electrical equivalent circuit model. The constructed model can be used with the circuit analysis program to predict the operating characteristics of the piezoelectric transformer in the target system. The optimal design procedure of the PT to meet the circuit operation requirements is also presented. The design procedure and the model was verified by experimental data using the prototype PT sample.

I. INTRODUCTION

Nowadays, more and more portable electronic products require ultra-slim power supplies, and the piezoelectric transformer (PT) technology provides a possible solution by replacing the bulky magnetic transformers with the low-profile PT as in Fig. 1.

Among the various kinds of PT structures, a disk-type transformer shows no spurious mode vibration, thus stable operation is obtained[1,2]. However, as its primary and secondary electrodes are stacked onto each other, its profile cannot be made lower with a high input voltage level corresponding to the line input voltage rating. In this research, a new disk type PT structure is developed as a main energy transformer in the AC/DC adapter. In this new structure, the primary and secondary electrodes are divided in concentric form. As compared with the conventional one, it has the advantage of simultaneously obtaining a low profile and a high withstanding voltage on the primary side, which is an especially suitable characteristic for the miniaturized AC/DC adapter application.

Once the PT structure is determined, the next step is to design its physical dimensions to meet the target system requirement. However, the PT involves both electrical and mechanical mechanisms, and thus the design and optimization of a piezoelectric transformer is a complicated engineering task that involves knowledge of physical acoustics theory and the properties of the materials. Without the aid of prediction tools for the resulting characteristics, the design of the PT according to the given circuit specification is very tedious and usually depends on a trial-and-error approach. In addition, the practical production of the pilot sample takes time and requires high cost. Currently, this pre-production design is usually processed iteratively by a mesh - based approximate solution using commercial finite-element tools such as ANSYS[4] or FEMLAB[5], which need considerable time to obtain solutions even for a single combination of data sets. Moreover, they give little design insight of how the structural dimensions are to be changed to meet the target system..

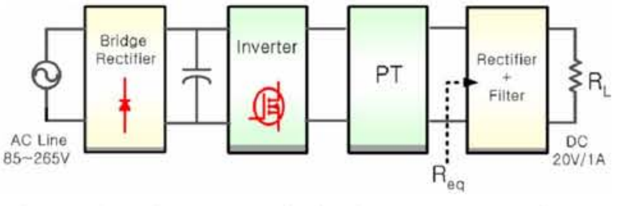


Fig. 1: AC/DC adapter system adopting the PT as a power transformer.

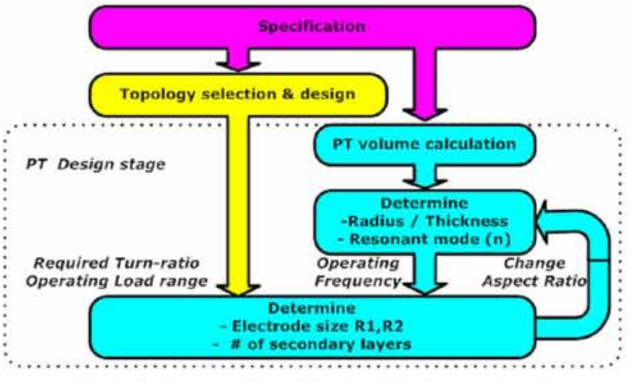


Fig. 2: Design procedure of the AC/DC adapter using PTs

On the contrary, the closed-form solution from the wave equation based on the acoustic physics of the piezoelectric transducer, can be used to give the design information for the piezoelectric transformer to meet the requirement of circuit specifications. It is especially valuable when the shape and the vibration mode of the PT are pre-determined by the production process, which is usually the case.

In this paper, the closed-form solution of the wave equation in the new disk-type PT was obtained by adding a mechanical dissipation term to the conventional lossless wave equation, and was incorporated to the electrical equivalent circuit. The derived model can also be used with the circuit analysis programs to predict the operating characteristics. Moreover, by determining the size of the

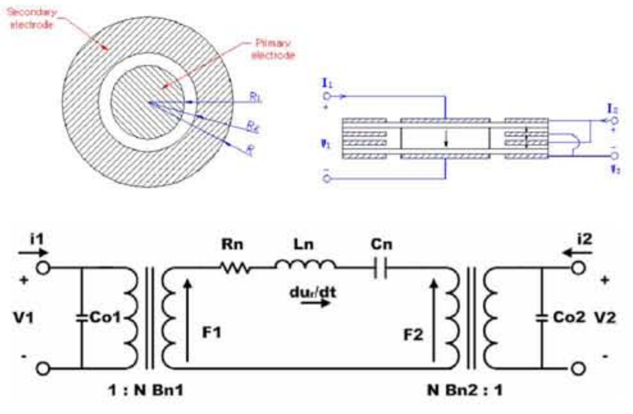


Fig. 3: Structure of the radial-mode disk-type PT used in this paper and its equivalent circuit.
(Arrows in the upper figure denote poling directions)

electrodes and the number of secondary layers in the PT design procedure to achieve a required voltage gain, the AC/DC adapter system with the PT can be optimally designed following the procedure shown Fig.2. For the purpose of verification, the prototype of the disk-type PT sample was constructed based on the design results and was compared with the empirical data established by measuring the prototype PT sample.

II. Radial-Mode Disk-Type Piezoelectric Transformer And Its Internal Wave Equations

The structure of the PT sample, produced for a main step-down transformer in a 20W AC/DC adapter, is shown in Fig.3. The fundamental resonant frequency is around 80kHz and the secondary mode is about 200 kHz. The primary electrode is placed in the center, and the secondary side consists of multiple layers.

Since the PT transfers electrical energy utilizing the mechanical vibration, the wave equation describing the particle displacement in the PT should be established to characterize its energy transfer mechanisms. For the radial-mode disk-type PT in the lossless case, this equation was obtained as in (1), where $u_r(r)$ is a phasor representing the particle displacement in the radial direction. The detailed derivation is in the Appendix.

To model the PT for power electronics applications, loss generation inside the PT operating with power circuits, should be taken into account. The energy dissipation is caused by internal friction due to the imperfect elasticity of the piezoelectric materials[3]. The frictional resisting force can be represented by equivalent viscous damping [9]. In this case, the damping force is proportional to velocity and is added to (1), resulting in the following modified version of the wave equation (2) where, κ is a proportional constant related to the frictional effect and can be determined from measurements.

A solution of (2) is obtained as in (3), where $J_1(r)$ is a Bessel function of the first kind [10], v_t is the wave propagation velocity, K_n is an nth mode eigenvalue, and ω_n is the associated eigenfrequency - all the symbols are defined in the Appendix. The quantity B_{np} is a measure of the coupling between the pth electrode and the nth mode.

From the electrical terminal point of view, the PT constructs a two-port network making the mechanical portion into a black box. In the next section, the solution $u_r(r)$ was used for the derivation of y-parameters describing the electrical terminal behavior based on two-port network theory.

III. The Equivalent Circuit Modeling

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{1}{r^2}u_r+s_t\rho\omega^2u_r=(1-\sigma)d_{31}\frac{\partial E_z}{\partial r},\quad E_z(r)=\begin{cases}V_p/\Gamma & (0\le r\le R_1, r\ge R_2)\\ 0 & elsewhere\end{cases} \tag{1}$$

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{1}{r^2}u_r+j\omega s_t\kappa u_r+s_t\rho\omega^2u_r=(1-\sigma)d_{31}\frac{\partial E_z}{\partial r} \tag{2}$$

$$u_r(r)=\left(\frac{-v_t^2(1+\sigma)d_{31}}{\omega^2-j\omega\left(\kappa/\rho\right)-\omega_n^2}\right)\left(\frac{2K_n^2}{K_n^2-1+\sigma^2}\right)J_1(rK_n/R)\left[\left(\frac{V_1}{\Gamma}\right)B_{n1}+\left(\frac{V_2}{\Gamma}\right)B_{n2}\right] \tag{3}$$

$$B_{n1}=\frac{R_1J_1(R_1K_n/R)}{R\ J_1(K_n)},\qquad B_{n2}=\sqrt{N_{sec}}\times\frac{R\ J_1(RK_n/R)-R_2J_1(R_2K_n/R)}{R\ J_1(K_n)} \tag{4}$$

$$I_p=j\omega C_{op}V_p-\left(\frac{\rho\dfrac{2\pi j\omega d_{31}^2}{(s_{11}+s_{12})^2}}{\omega^2-j\omega\left(\kappa/\rho\right)-\omega_n^2}\right)\left(\frac{2K_n^2B_{np}}{K_n^2-1+\sigma^2}\right)\left[\left(\frac{V_1}{\Gamma}\right)B_{n1}+\left(\frac{V_2}{\Gamma}\right)B_{n2}\right] \tag{5}$$

$$Y_{pq}=\frac{\partial I_p}{\partial V_q}=j\omega C_{op}\delta_{pq}-\left(\frac{\dfrac{4\pi j\omega d_{31}^2}{(s_{11}+s_{12})^2}}{\omega^2-j\omega\left(\kappa/\rho\right)-\omega_n^2}\right)\left(\frac{K_n^2B_{np}B_{nq}}{K_n^2-1+\sigma^2}\right)\frac{1}{\rho\Gamma},\ \delta_{pq}=\begin{cases}1 & p=q\\ 0 & p\ne q\end{cases} \tag{6}$$

A. Admittance matrix (y-parameter) derivation

To calculate the electrical self-admittance and transfer-admittance [11] between the electrodes, the current at the p[th] electrode was obtained as a time derivative of the total electric displacement on the electrode[8],

$$D_z = d_{31}T_{rr} + d_{31}T_{\theta\theta} + \varepsilon_{33}E_z,$$
$$I_p = j\omega \int_{electrode\ p} D_z(r)2\pi\, rdr \qquad (7)$$

Solving (3) and (7) gives the electric current (5) where, C_{op} is the capacitance of the p[th] electrode defined by

$$C_{o1} = \varepsilon_{33}\frac{\pi R_1^{\ 2}}{\Gamma},$$
$$C_{o2} = N_{sec} \times \varepsilon_{33}\frac{\pi(R^2 - R_2^{\ 2})}{\Gamma/N_{sec}} \qquad (8)$$

Differentiation of I_p with respect to V_q gives y-parameters as in (6).

B. Equivalent lumped parameter model

If an equivalent circuit model[6] for the PT is introduced as in Fig. 2, the lumped parameter in the vicinity of the nth resonance can be calculated from (6) as follows.

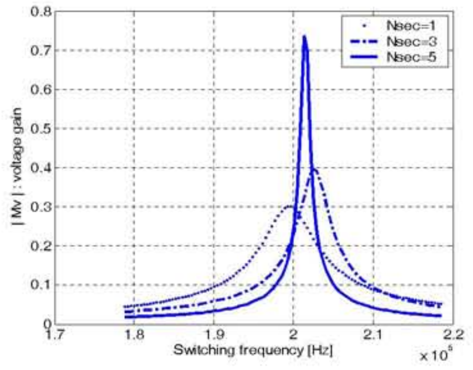
$$L_n = \frac{\pi R^2\Gamma\rho\left(K_n^{\ 2} - 1 + \sigma^2\right)}{K_n^{\ 2}},$$
$$C_n = \frac{s_t}{\pi\Gamma\left(K_n^{\ 2} - 1 + \sigma^2\right)}, \qquad (9)$$
$$R_n = \frac{\kappa}{\rho}L_n,$$
$$N_o = \frac{2\pi R d_{31}}{s_{11} + s_{12}}$$

This circuit model can be used to simulate the overall AC/DC adapter circuit, including the PT, by commercial circuit analysis tools such as PSpice or PSIM. The derived circuit model is also used in the optimal design process as presented in section IV.

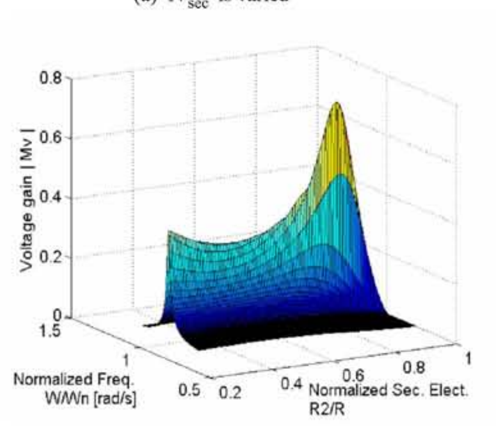
C. Derivation of the design equation from the model

When the operating frequency (ω) of the power stage is near the n[th] resonant frequency (ω_n) of the PT, the voltage gain or turn-ratio of the PT with an equivalent load impedance R_{eq} is then obtained using (8) and (9). The gain function depends on both the operational variables(ω , R_{eq}) and the dimensional variables (R, R_1,R_2,and N_{sec}) as shown in Fig.4.

$$M_V(\omega, R_{eq}) = \frac{V_2}{V_1} = \frac{Y_{21}}{\frac{I_2}{V_2} - Y_{22}} = \frac{\frac{\partial I_2}{\partial V_1}}{-\frac{1}{R_{eq}} - \frac{\partial I_2}{\partial V_2}} \qquad (10)$$



(a) N_{sec} is varied



(b) R_2 is varied

Fig. 4: 2nd mode gain characteristics of the PT with dimensional parameter as a variable when (a) R =15, R_1 =4.5, R_2 =6.5[mm], R_{eq} =100[Ω], (b) R =15, R_1 =4.5[mm], N_{sec} =3, R_{eq} =100[Ω]

By choosing the dimensional variables to achieve a required voltage gain using (10), the AC/DC adapter system with the PT can be optimally designed as shown in Fig.1.

IV. DESIGN OPTIMIZATION OF THE PT FOR AC/DC ADAPTER

As mentioned in the previous section, the derived electrical terminal behavior can be used to evaluate the performance of the PT for the target system.

A prototype 20W AC/DC adapter shown in Fig.5 consists of a boost pre-regulator which provides a dc-link voltage, a frequency-controlled half-bride topology [15] used to drive PT and to regulate output voltage, and a current-doubler rectifier to obtain the DC output voltage from the PT. The specifications of the target system are :

- AC input voltage : 85~265 [Vrms] 60Hz
- DC-link voltage : 375 ± 10 [Vrms]
- Regulated output voltage : 20 V
- Maximum output current : 1 A

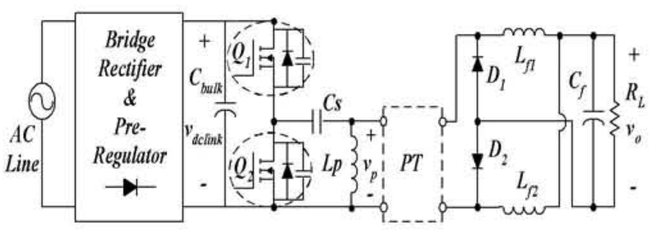


Fig.5: A prototype of AC/DC adapter using the PT. The derived PT equivalent circuit is further simplified and incorporated into the overall circuit[15].

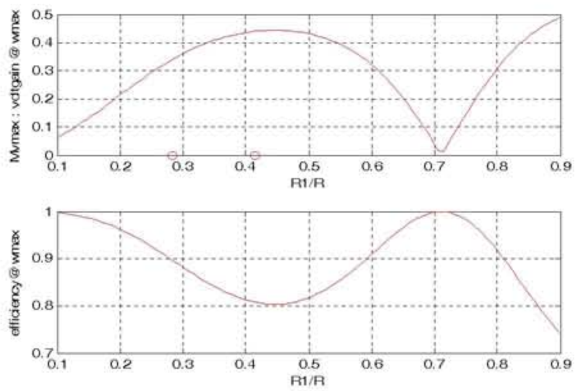


Fig. 6: 2[nd] mode peak voltage-gain characteristics of the PT with R1 as a design parameter. R2 is predetermined by optimal load condition.

A. Step 1: Determination of overall radius R of the PT.

The power volume capacity of the current PT material (modified PZT-4 from Dong-il Tech) is experimentally estimated to be about 1.6 [Watt/gram], and a sufficiently large aspect ratio (R/Γ) of about 5 is required to ensure the radial mode operation approximation in the thin disk structure. The total radius of the disk structure is determined from the handling power of the PT as,

$$R = \sqrt[3]{\frac{Aspect\_ratio \times P_o/\eta_{rect}}{1.6\times 10^3 \pi\rho}} \,. \tag{11}$$

In this application R=15 [mm] was obtained.

B. Step 2: Calculate R2 from the optimal load condition.

A PT should be designed to provide maximum efficiency at the full load condition. By the fundamental frequency approach suggested in [13], an ac equivalent load of the PT, including the rectifier, was obtained as,

$$R_{eq} = \frac{\pi^2}{2}(1+\frac{V_F}{V_o})^2 R_L \tag{12}$$

where V_F is the diode forward drop and V_o is the output dc voltage. From the equivalent circuit analysis [14], maximum efficiency resonant peak point can be achieved when the load resistance is

$$R_{opt} \approx \frac{1}{\omega_{rs}C_{o2}} = \frac{\sqrt{L_n C_n}}{C_{o2}} \tag{13}$$

where ω_{rs} is the resonant frequency of the PT when the secondary port is a short.

From (8) and (13), the PT can be designed by adjusting N_{sec} and/or the R_2 value to have optimal load matching near an R_{eq} of about 100 ohm, which is the calculated value in this application. First of all, if N_{sec} is selected as an integer value, the R_2 value is directly given by

$$R_2 = \sqrt{R^2 - \Gamma(\omega_{rs}R_{eq}\pi\varepsilon_{33}N_{sec})^{-1}} \tag{14}$$

C. Step 3: Determine R1 from the peak gain curve

With this R_2 value, R_1 can be designed for the PT to have a voltage gain required for the target system. Differentiating (10) with ω and equating it to zero gives the peak voltage gain equation as,

$$M_{v,peak} = \frac{\frac{B_{n1}}{B_{n2}}\sqrt{1+\frac{Q^2}{2}\left((1-\frac{1}{Q^2})+\sqrt{(1-\frac{1}{Q^2})^2+\frac{4}{Q^2}}\right)}}{1+\frac{c}{Q_m Q}\left(1+\frac{Q^2}{2}\left((1-\frac{1}{Q^2})+\sqrt{(1-\frac{1}{Q^2})^2+\frac{4}{Q^2}}\right)\right)}$$

$$\text{where } Q=\omega_{rs}R_{eq}C_{o2}, Q_n = \frac{1}{\omega_{rs}R_n C_n}, c = \left(\frac{B_{n1}}{B_{n2}}\right)^2\frac{C_{o2}}{C_n}. \tag{15}$$

At this peak point, efficiency is given by

$$\eta_{@Mv,peak} = \frac{1}{1+\frac{c}{Q_m Q}\left(1+\frac{Q^2}{2}\left((1-\frac{1}{Q^2})+\sqrt{(1-\frac{1}{Q^2})^2+\frac{4}{Q^2}}\right)\right)}$$

There is another constraint in the design of R_1. There should be a minimum insulation distance of about 2 mm between the primary and secondary electrodes to survive the insulation test, and thus a possible R_1 value is,

$$R_1 \le R_2 - \min.gap \,. \tag{17}$$

From the specification of the target system, the required gain of the PT in the maximum load condition is calculated as,

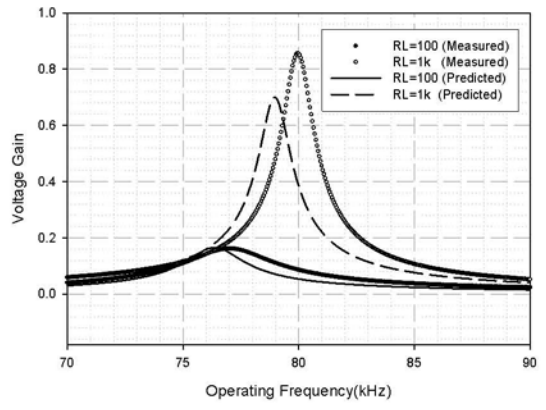
$$M_{v,required} = \frac{\sqrt{R_{eq,\max}\frac{P_{o,\max}}{\eta_{rect}}}}{\frac{\sqrt{2}}{\pi}\frac{\sin(\pi D_d)}{\pi D_d}\times V_{dc-link}}\times k = 0.32 \tag{18}$$

where D_d is dead time duty defined in [15], and k is a design margin factor of the peak value, which in this case, a value of about 1.1 is sufficient, thus providing a 10% margin.

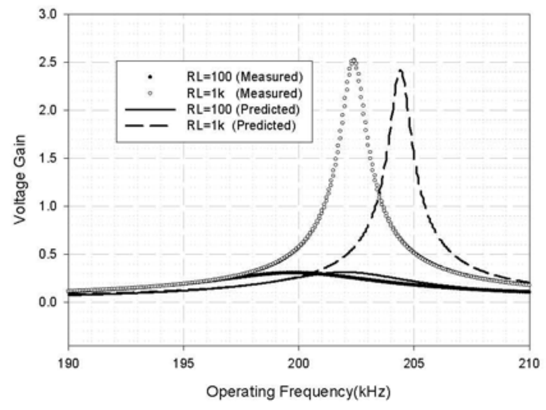
Considering the possible range of R_1, the peak voltage gain vs R_1 is shown in Fig. 6. Among the possible sets which meet the required gain condition above, $N_{sec} = 3$, $R_1/R = 0.3$, $R_2/R = 0.4$ achieves the best efficiency and was shown in Table II.

V. EXPERIMENTAL VERIFICATION

Prototypes of the disk-type PT sample was produced based on the optimal design and the empirical equivalent circuit modeling was performed by measuring the admittance using an HP4194A network analyzer and adopting a complex curve-fitting algorithm[12]. Then circuit simulations were performed using the empirical model. Based on the material constants and the dimensional data of the piezoelectric transformer, shown in Table I, an admittance matrix (6) was constructed and voltage gain versus the operating frequency with a given load, R_{eq} , was calculated using (10). Fig. 7 shows the compared results of the theoretical and empirical voltage gain curves, which verifies the derived results.



(a) 1st resonant mode



(b) 2nd resonant mode

Fig. 7: Comparisons of gain curve characteristics of the PT with resistive load condition R_{eq}=100[Ω] (heavy load) R_{eq}=1[$k\Omega$] (light load)

VI. CONCLUSIONS

The proposed modeling of the radial-mode disk-type PT based on the closed-form solution of the wave equation provides a powerful method to derive the design information for the AC/DC adapter adopting a PT. The empirically verified terminal behavior constructed an electrical equivalent circuit to be used in time simulation with a common circuit analysis program. Based on the proposed design procedure, the optimal design of the PT as a circuit component in the power converter is also presented.

APPENDIX

A. Derivation of the wave equation in loss-less case [8]

In an angularly symmetric piezoelectric thin disk, the constituent relations, according to the IRE standards[7], are represented in the polar coordinate system as,

$$S_{rr} = s_{11}T_{rr} + s_{12}T_{\theta\theta} + d_{31}E_z$$
$$S_{\theta\theta} = s_{12}T_{rr} + s_{11}T_{\theta\theta} + d_{31}E_z \qquad (19)$$

Strain on a particle of the PT is obtained in this case

$$S_{rr} = \frac{\partial u_r}{\partial r}$$
$$S_{\theta\theta} = \frac{1}{r}u_r \qquad (20)$$

where u_r (r) is a phasor representing the particle displacement. Newton's equation is

$$-\rho\omega^2 u_r = \frac{\partial T_{rr}}{\partial r} + \frac{T_{rr} - T_{\theta\theta}}{r} \qquad (21)$$

Substitution of (19) and (20) into (21) then yields (1).

Table I. Material constants of the PT sample

Material constants (PZT-4)		
Density	ρ	7500 [kg/m3]
Compliances	s^E_{11}	12.0 x 10-12 [m2/N]
	s^E_{12}	-3.84 x 10-12 [m2/N]
Piezoelectric constants	d_{31}	-122 x 10-12 [C/N or m/V]
Permittivity	ε^T_{33}	1.16 x 10-8 [F/m]

Table II. Dimensions of the prototype PT sample

Dimensions of the PT sample		
Total radius of the disk sample	R	15.2 [mm]
Radius of the primary electrode	R_1	4.50 [mm]
Radius of the secondary electrode	R_2	6.50 [mm]
Thickness of the disk	Γ	2.54 [mm]
Number of Secondary Layers	N_{sec}	3